Fractal Deviation: Implication to non-verval Reasoning and Students' Mathematics Competency

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Abstract - *Fractals are geometric shapes with very interesting characteristics that beyond typical Geometry. In this study, it was hypothesized that fractal deviation of students' handwriting skills is significantly correlated to their non-verbal reasoning and mathematics performance. Fifteen Grade 10 high school students served as participants of the study in which their handwriting skills were analyzed through copying the prototype drawing (FD = 1.2837). The samples of students' drawing were subjected to fractal dimension analyses to find the fractal deviation. Pearson's correlation was used to test the significant relationship among variables. After analyzing the result, it was found that fractal deviation of students' handwriting skill is significantly related to their mathematics competency. Likewise, students who are mathematically inclined have higher fractal deviation index considering their handwriting than those who are not. Also, it was concluded that the higher the non-verbal reasoning skills of the students, the higher their Mathematics competency. Finally, no significant relationship was established between fractal deviation of students' handwriting skills and their non-verbal reasoning.*

Keywords – Fractal Deviation, Mathematics Education

INTRODUCTION

Fractals are geometric shapes with fascinating properties that distinct from typical Euclidean shapes (points, lines and planes). Its self-similarity property implied that geometric object is similar in both a qualitative and quantitative sense over many different measurements. Historically, after Fractal Geometry was credited to Benoit Mandelbrot (1975), voluminous findings on the applications of fractals have been presented and published [1]. For instance, fractals are used to detect forged handwriting using a fractal number estimate of wrinkliness [2]. Fractals are found also to be useful for cancer discovery and the description of pathological construction of tumors using their (fractals) irregularity structure [3]. In technology, fractals are considered as one of the main contributors in the present day computer graphics. The complexities in the system as well as in the combinations of colors can be attributed to fractals [4]. The timing and magnitude of earthquakes and the variation in a person's heartbeat and the occurrence of diseases are some of many cases in which fractal can describe the complication [5]. Regardless of its complexity, fractals are now being studied and included in the present day curricula. The Mathematics in the Modern World as part of the general education curriculum begins with an introduction of mathematics as an investigation of patterns in nature and appreciating geometric designs in the environment [6].

In the field of Education, lots of predictors of students' math performance were already identified such as profiles, motivations, anxiety, parents' involvement [7]-[10]. These predictors can positively or negatively influence the mathematics performance of the students. However, no concrete implication of nonverbal representation to mathematical performance of students is established yet [11]. Hence, this research applied the fractal geometry to link the gap between non-verbal reasoning and students' mathematics competency.

OBJECTIVES OF THE STUDY

This study was conducted to analyze the implication of fractal deviation (consistency in handwriting) to non-verbal reasoning and students' mathematics competency.

Specifically, it aims to:

1. determine the fractal dimension of students' consistency in handwriting when grouped according to their mathematical competencies;
2. identify the non-verbal competency of the fourth year students and their mathematics competencies; and
3. test the relationship between the fractality of students' handwriting; their academic performance in mathematics; and non-verbal competency.

FRAMEWORK OF THE STUDY

The conceptual framework of the study shows the relationship between non-verbal reasoning and mathematics competency of students. It likewise shows that both non-verbal reasoning and mathematics competency are both influenced by the fractal deviation of students' handwriting skills which is hypothesized to be a factor of having high or low variations.
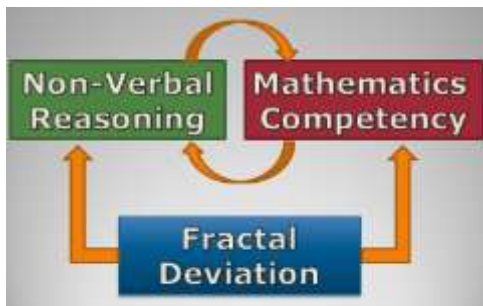

Figure 1. Conceptual Framework of the Study showing the interplay of the variables

MATERIALS AND METHOD

The present study is anchored on the theory of Mandelbrot [12] that Fractal Geometry plays two roles. It is the geometry of complexity and the geometry from mountains, clouds and chaotic galaxies. The researcher believes that the presence of patterns is not only present in the nature around people but also within people themselves. Accordingly Padua [13] stated that there was a new development in research using Fractal analysis and Statistics which do not rely only on central tendencies but more on geographical presentation of certain variables that researchers consider in dealing with research. Guided by these literatures and the descriptive design in research, the researcher purposively selected 15 fourth year (Grade 10) high school students with the help of their corresponding advisers. Their non-verbal reasoning was tested using the standardized test which was tested and validated by the experts in the field of Mathematics in a one University in the Philippines. Likewise, students were categorized into three groups: A, B and C which corresponds to their academic performance in mathematics. After the non-verbal reasoning test, they were requested to copy the given prototype (F_D=1.2837) ten (10) times using their own handwriting skills. Then, the researcher converted the copied drawing to electronic gadget and converts it to bitmap format for fractal analysis. The generated fractal dimensions were likewise collected and the fractal deviation per handwritten work was calculated. Finally, the inferential statistical analysis was done to find the relationship between their mathematics performance and their non-verbal reasoning as affected by the consistency in their handwriting.

RESULT AND DISCUSSION

The fractal deviations of students' handwriting, non-verbal reasoning as well as mathematics competency are presented in the succeeding tables. In addition, the relationships between them were analyzed to further strengthen the significance of the results. It can be seen from the table that the average fractal dimensions of Groups A, B, and C are 1.1116, 1.1772 and 1.2894 consecutively. Group A, having the most deviated dimension among the groups comprised 17.21% from the original dimension of the

prototype. This means that students from group A are characterized as students with the highest fractality dimension. On the other hand, the least fractal deviation is found in the fractal dimension of Group C which is 1.2894 (2.99% deviated from the original prototype). This means that Group C has the lowest fractal deviation among the three groups of respondents.

The percentage among the fractal deviations of students' handwriting skills was found significantly different from each other when classified according to respondents' groups. The average fractal deviation (in percent) in Group A is significantly higher than of Groups B and C while the average fractal deviation (in percent) of Group B is significantly higher than Group C.

Table 1. Fractal dimension and deviation of students' handwriting skills when grouped according to their mathematical competencies

| Classa	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	Average
					Fractal Dimension						
A	1.1059	1.1082	1.1096	1.111	1.1124	1.1138	1.1152	1.1166	1.1180	1.1059	1.1116
B	1.1714	1.1737	1.1751	1.1765	1.1779	1.1793	1.1807	1.1821	1.1835	1.1714	1.1772
C	1.2413	1.2136	1.2752	1.3045	1.3117	1.3129	1.2998	1.309	1.3121	1.3136	1.2894
					Fractal Deviation						
Class	**FD1**	**FD2**	**FD3**	**FD4**	**FD5**	**FD6**	**FD7**	**FD8**	**FD9**	**FD10**	**Average**
A	17.78%	17.55%	17.41%	17.27%	17.13%	16.99%	16.85%	16.71%	16.57%	17.78%	17.21%[a]
B	11.23%	11.00%	10.86%	10.72%	10.58%	10.44%	10.30%	10.16%	10.02%	11.23%	10.65%[b]
C	4.24%	7.01%	0.85%	2.08%	2.80%	2.92%	1.61%	2.53%	2.84%	2.99%	2.99%[c]

F = 477.765   P-value = <0.001
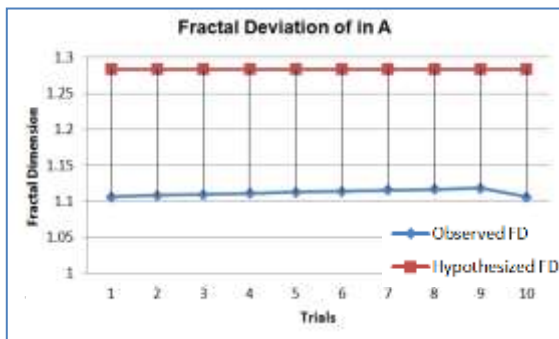
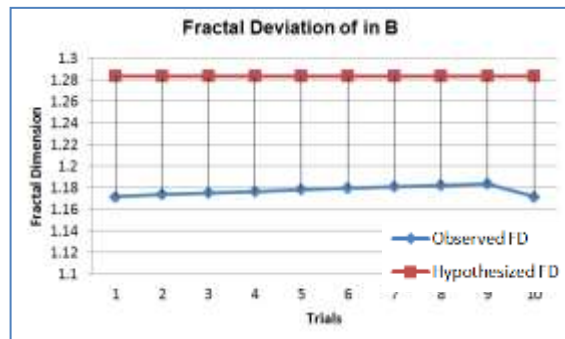*Figure 2. Fractal Deviation in Group A*
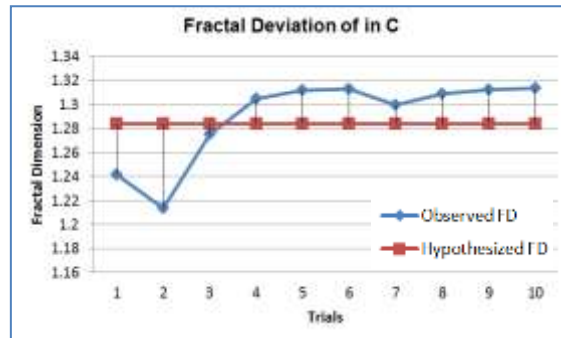

Figure 3. Fractal Deviation in Group B

Figure 3. Fractal Deviation in Group C

As depicted in table 2, Group A found to have the highest score in the non-verbal reasoning test while group C found to have the lowest score. However, it can be seen also that only class A passed the non-verbal reasoning tests. This means that in general, students have low reasoning skills.

Table 2. Non-Verbal Competency of the Fourth Year Students Considering their Mathematics Competencies.

Class	Non-Verbal	Percentage	Verbal Interpretation
A	10.8	77.60	Passed
B	8.2	74.57	Failed
C	7.4	73.63	Failed

As seen on the table below, no significant relationship existed between fractality of students' handwriting skills and their non-verbal reasoning skills as seen on the p-values 0.773 which is greater than the threshold value of 0.05. On the other hand, a significant relationship existed between the fractality of students' handwriting skills and their academic performance in mathematics. This means that the higher the deviation of students' handwriting skills the higher the students' performance. This result is parallel to what Henderson and Green [14] claimed that both highly intelligent and verbally fluent students may have a tendency of having poor handwriting skills. In addition, the findings agreed to what James and Engelhardt [15] claimed that writing letters by hand activated areas of child's brain (reading circuit). The implication of handwriting to mathematics performance can be established on the foregoing findings as supported by literatures presented since solving mathematical problems needs both the lower order thinking skills (remembering, understanding, applying) and higher order thinking skills (analyzing, evaluating and creating).

Table 3. Significant Relationship between the Fractality of Students' Handwriting to their Academic Performance in Mathematics; and Non-Verbal Competency.

FD vs.	Mean	Stat.Tool	r-value	r-sqrd	p-value	Relationship
Academic Performance	83.81	Pearson Correlation	0.603	36.3609%	0.017	Significant
Non-Verbal Reasoning	10.2		-0.081	0.6561%	0.773	Not Significant

Non-verbal reasoning and students' academic performance in mathematics are significantly related as seen on the p-values 0.008 leading to the rejection of the null hypothesis. In addition, the coefficient of determination 18.23% signifies that 18.23 percent of the variation in non-verbal reasoning is attributed to students' mathematics competency. This implies that the higher the non-verbal reasoning skills of the students, their mathematics competency gets increase. The findings conforms to the study conducted by Drager [10] that the performance on an algebra final exam is related to abstract reasoning after controlling for age, motivation, and previous math achievement of the students. Results from these analyses indicate that abstract reasoning ability is the most important among the variables studied in predicting success in high school algebra. Scores on the Abstract Reasoning Assessment predicted performance in a high school algebra course even after controlling for general mathematics ability. The findings suggest strongly that non-verbal reasoning ability is critical for success in mathematics.

Table 4. Significant Relationship between the Students' Academic Performance in Mathematics and Non-Verbal Competency.

Variables	Mean	Stat.Tool	r-value	r-sqrd	p-value	Relationship
Academic Performance	83.81	Pearson Correlation	0.427	18.23%	0.008	Significant
Non-Verbal Reasoning	10.2					

CONCLUSION

Based on the findings presented, high extent of inclination to non-verbal reasoning will result to better performance in mathematics. More so, students who are mathematically inclined have higher fractality index considering their handwriting than those who are not signifying that students who have poor handwriting performance have greater chance of having good mathematical thinking skills. Applying the concept of Geometry, through Fractals, in analyzing the connections between variables is considered as success in determining the connection between the theoretical foundations of Mathematics and a more sophisticated real world of learning. Likewise, the variables correlated provided remarkable results as they showed significant inferences. Hence, the study showed that fractal has an implication to educational learning status of the students.

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